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## **OTC Derivatives for Retail Investors**

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## **Abstract**

In this paper we report on a new class of derivative products which we refer to as equity-linked savings products. Equity-linked savings products require investors to pay periodic instalments in return for a predefined equity-linked payoff at maturity. We discuss the structuring, hedging, pricing and marketing of a variety of equity-linked savings products in detail. We pay particular attention to the case of The Netherlands where equity-linked savings products are currently very popular with an estimated USD 2 billion issued over the last three years. Reverse engineering shows that the profit margins which product providers may be able to achieve on equity-linked savings products are extremely high.

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\*I would like to thank participants of the 7<sup>th</sup> Multinational Finance Society Conference in Philadelphia and the Financial Options Research Centre Summer Seminar at the University of Warwick for helpful comments.

## **I Introduction**

Stock Market investing has never been as popular in Europe as it is in the US but the recent bull market has inspired even retail investors in Europe to look for ways to obtain equity exposure. Apart from direct investment, many European investors have bought so-called principal protected notes. These are medium term (typically zero coupon) notes that offer equity exposure without the risk of losing one's principal. In other words, the interest paid on these notes is linked to the performance of one or more reference indices. If the index goes up, the interest goes up. If the index goes down, the interest goes down but never becomes negative. Principal protected notes have become very popular with retail investors in Europe where the estimated total issuance from the beginning of 1997 until the end of the first half of 1999 alone was over \$80 billion. In the U.S. principal protected notes have been less successful so far, indicating that U.S. retail investors are much more comfortable with direct equity investment and mutual funds than their European counterparts.

The level of equity market participation offered by the typical principal protected note has come down a lot over the last five years. A principal protected note is able to offer equity exposure by using the interest that would normally be paid by the issuer to buy call options with. As a result, the participation rate depends heavily on interest rates and the market price of call options. The lower the interest rate, the less money there is to buy calls with. The more expensive the options the less we can buy. When interest rates fall and at the same time options become more expensive the participation rate comes down very quickly. This is exactly what has happened over time. In 1994-1995 interest rates were relatively high and implied volatilities

relatively low, in some cases allowing for notes that paid their holders more than 140% of the positive return on the index. By the end of 1995, however, participation rates on all major indices started a downward trend that lasted for almost four years. Apart from a rise in option prices an important factor was the decline in interest rates. EURO participants saw their longer-term rates converge to 3.5-4.0% with especially Spain and Italy showing accelerated rate cuts during the second half of 1998. By the end of 1998 participation rates on most major European indices had dropped to 70% or less. In the beginning of 1999 participation rates picked up again due to falling long-dated implied volatility and a slight rise in longer-term rates. With participation rates coming, principal protected notes have become less interesting investment vehicles. This is reflected in not issuance activity. According to estimates from Warburg Dillon Read<sup>1</sup>, average issuance of longer-dated principal protected structures in Europe in 1997 was about \$10 billion per quarter. Although in part due to changes in the tax and regulatory environment, in 1998 this dropped to \$7.7b, while in 1999 it dropped further to \$4.8b.

Apart from the falling participation rate there is another problem with

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<sup>1</sup> See Ineichen (1999).

principal protected notes: they are aimed at investors that already have money. This leaves a large group of people that would like to invest in equity but who simply do not have enough cash available (yet). Although the direct purchase of investment products is not an option, there is an alternative way for people like this to obtain the equity exposure they desire. They can enter into a swap contract with a product provider (typically a bank or insurance company) where over some period of time they pay the latter periodically, say monthly, a prefixed amount and in return receive

an index-linked payment at maturity. In this way they obtain equity exposure without making an upfront payment.

In what follows we discuss a number of such equity-linked savings products in more detail assuming that the investor makes monthly payments of 100 over a period of five years. The relevant reference index is assumed to be at 100 and pays no dividends except where mentioned. We assume that the derivatives firm which is asked to price the contracts in question uses term structures of interest rates and implied volatility which are flat and constant at 5% and 20% respectively and that the prices thus obtained include the derivatives firm's profit margin. Time is measured in years. Present time is denoted as time  $t = 0$ , five years from now as  $t = 5$ , etc. The value for the reference index at time  $t$  is denoted as  $I_t$  and the continuously compounded interest rate as  $r$ .

## **II Unprotected Equity-Linked Saving**

With the investor making 60 monthly payments of 100, the question is what equity-linked payment at maturity the product provider to take the investor's money every month and invest it in some stock or stock portfolio as soon as it comes in. At maturity the product provider would liquidate the portfolio and pay the proceeds to the investor. The problem with this buy-as-you-go scheme is that the investor does not know in advance what the payoff is going to be. If the reference index goes up, the product provider can buy the investor less shares and if it goes down he can buy him more. Of course, the product provider need not necessarily buy stocks. If he wanted a less risky product he could buy principal protected notes. If he wanted a

more risky product, he could buy forwards or ordinary call options. All these alternatives, however, suffer from the same deficiency: the investor can not tell what the payoff is going to be because he does not know the prices he will have to pay in the future. Another problem with the buy-as-you-go products is that the exposure builds up very slowly over time. For investors who want instant exposure, products like these are not a very attractive alternative. In this section we therefore discuss two products that offer the investor significant exposure directly from the outset.

Suppose the investor wanted to invest in a single reference index that paid no dividends. This reference index could be a single stock or a market index but also a fund participation. In that case the product provider could offer the investor a payment at maturity equal to

$$X_5 = M \times I_5, \quad (1)$$

Where the multiplier  $M$  is a constant. This payoff looks very much like the payoff of a buy-as-you-go scheme but there is one important difference: the multiplier is known in advance. We will refer to this payoff as **product 1**. With the investor only paying the product provider 100 every month, it is intuitively clear that the multiplier in (1) can not be very large. If it is just exposure the investor is after the product provider can do a lot better by introducing some leverage. He could for example offer the investor a payoff equal to

$$X_5 = M \times I_5 - N_5, \quad (2)$$

where the terminal lump sum  $N_5$  is a fixed amount. We will refer to this payoff as **product 2**.

To be able to say something about the multiplier of product 1 of the multiplier and the terminal lump sum of product 2 we have to understand how the product provider, which will typically be a bank or insurance company without any derivatives capability of its own, is going to hedge himself. The most obvious way is for the product provider to approach a derivatives firm to enter into a swap like the one with the investor and simply pass all the payments through (after taking out a profit margin). The investor pays the product provider 100 per month for five years. Suppose the latter takes a profit of 5.62 out of every 100 for himself, and passes the remainder through to the derivatives firm. The derivatives firm therefore has to be determined such that the index-linked payment to be made by the derivatives firm at maturity is worth 5,000. Taking a closer look at the latter payment, we see that it is nothing more than the payoff of  $M$  index participations. For simplicity assuming the firm hedges itself in the cash market and abstracting from transaction costs, tracking error, etc., to hedge one index participation the derivatives firm will need to invest an amount of  $I_0 = 100$  in the index. Since it has 5,000 available, this means that it can offer a multiplier of 50. In other words, in return for 60 monthly payments of 100 the investor will receive the value of 50 shares of the index five years from now.

We can do the same for product 2. The derivatives firm receives a stream of coupons with a present value of 5,000. In return it has to pay an amount of  $M \times I_5 - N_5$  at maturity. This amount consists of two parts. As before, the first part is the payoff of  $M$  index participations. This can be hedged by the purchase of  $M$  shares in the index which currently trades at  $I_0$ . The second part can be hedged by borrowing an amount equal to  $e^{-5r} N_5$  as this will create a debt of  $N_5$  at maturity. The proceeds of the loan

can be used to buy additional index participations. The budget equation for product 2 is therefore given by

$$5,000 = M \times I_0 - e^{-5r} N_5 \quad (3)$$

The budget equation for product 2 leaves some freedom as to the values of the parameters involved. The product provider can choose the terminal lump sum and use expression (3) to find the corresponding value of the multiplier, or he can choose the multiplier and solve for the terminal lump sum. From an economic point of view there are definite limitations to this though as the payoff given by expression (2) need not be positive, i.e. the investor may end up paying the product provider at maturity.

Figure 1 shows the terminal lump sum as a function for the multiplier chosen. If the product provider set  $M = 50$  the terminal lump sum equals zero, which brings us back to product 1. Since  $I_5$  will always be equal to or larger than zero, the only risk the product provider runs here is that the investor fails to make his monthly payments. This changes if we move on to higher multiplier values. With a multiplier higher than 50, the terminal lump sum will be higher than zero. Since the value of the equity part

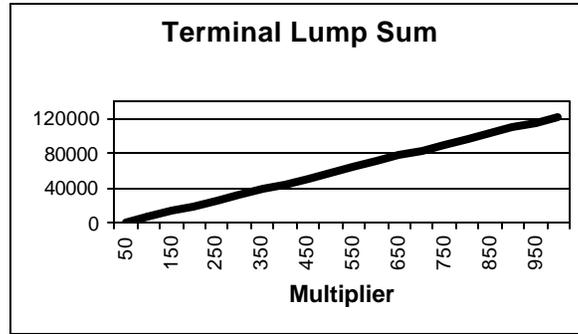


Figure 1: Terminal lump sum as function multiplier

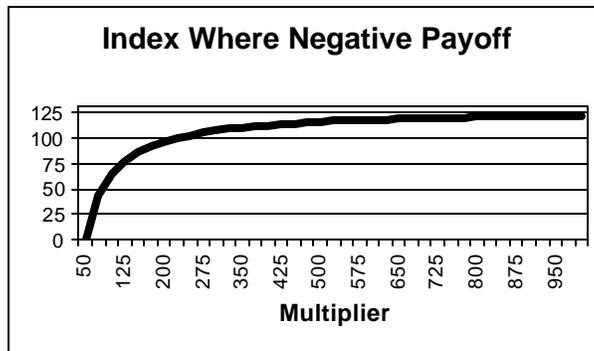


Figure 2: Index value below which payoff product 2 is negative as function multiplier.

of the payoff can theoretically drop to zero, the investor is now confronted with the risk of a negative payoff. This risk increases with the multiplier. If the product provider set  $M = 500$  the terminal lump sum would be equal to 57,781. This means that product 2 provides the investor with a negative payoff if at maturity the reference index is below 121.98. How the index value below which the payoff of product 2 is

negative varies with the multiplier is shown in figure 2. If the multiplier is raised, the index value below which the payoff of product 2 is negative converges to an index level of 128.4.

An interesting choice is to set the multiplier such that  $N_5 = M \times I_0$ , meaning that at maturity the investor receives a payoff equal to  $M \times (I_5 - I_0)$ , i.e.  $M$  times the change in the value of the index. Doing so yields a multiplier of 226. In other words, by making 60 monthly payments of 100 the investor acquires a payoff of 226 times the change in the value of the index over the next five years. We can calculate the investor's return as a function for the index value at maturity if we relate the product payoff to the present value of his monthly payments (which is 5,298). The result is shown in figure 3. If the index rises by 60% the investor receives an amount of 13,560. This represents a return of 156%. If the index goes up by 100%, the investor makes 327%. Throughout the remainder of this paper we will concentrate on the above version of product 2.

With a multiplier of 226, if the index drops by 10% the investor owes the product provider an amount of 2,260. The present value of 5.62 out of every 100, however, is only 298. This brings up the question whether taking 5 out of every 100 is enough to compensate the product provider for the credit risk he is taking. In answering this question we have to keep two points in mind. First, the probability that the reference index shows a drop over a five-year period is relatively small. With an expected annual index return of 10% and an annual index volatility of 20%, the probability of a 5-year return lower than zero is only about 13%. Second, we are talking about a retail product here. If successful, the product will be sold to at least tens of thousands of

investors. Although they will all lose if the index drops, most of them can be expected to pay up. In other words, the credit risk is well diversified.

In the context of credit risk it is interesting to note that with  $N_5 = M \times I_0$  the multiplier in product 2 is independent of the maturity of the product<sup>2</sup>. With a 6-year maturity the product provider would also have ended up with a multiplier of 226. This means that in case the index drops and the product payoff is negative he can offer the investor to extend the contract at no extra cost. The investor could continue to pay his monthly coupons and at the new maturity date he would still be paid 226 times the change in the index. The mirror image of the right to extend a contract is the right to make it mature prematurely. The product provider could therefore also structure a long-dated contract and offer the investor the possibility to exercise the contract early.

### **III Dividends**

So far we have assumed that the reference index does not pay dividends. If it does, this significantly changes the pricing of product 1 and 2. Since the product only pays the value of the index at maturity, the dividends are left with the derivatives firm.

This raises the amount received by the latter and therefore allows it to quote a higher multiplier. Suppose the reference index paid a fixed annual dividend of 3.00 per share. The present value to the derivatives firm of all dividends received on the reference index over the product's life would in that case be equal to 12.95 per share of the index. This means that it can generate an amount of  $I_5$  at a cost of  $I_0 - 12.95 = 87.05$ , which yields a multiplier of a little over 57; a significant improvement over the

<sup>1</sup>50 we had before. We can do the same for product 2 (with  $M \times I_5 - I_0$ ). This yields a multiplier equal to a staggering 545, compared to 226 before. Figure 4 shows the multiplier as a function of the annual dividend paid by the index. Many retail investors have little preference when it comes to stock selection other than that they want well-known names. Since stocks that pay relatively high dividends allow for a higher multiplier this allows the product provider to beef up product 1 and 2 by using a basket of exactly those stocks as reference index.

Of course, future dividends are not known with certainty. Since companies tend to be reluctant to reduce dividends, however, the assumption that future dividends will at least equal last year's dividend seems to carry little downside risk. Another way to deal with dividend risk is to let the investor absorb it himself. In other words, if dividends do not turn out as expected at the time the product was priced the investor will be asked to pay the difference. In Australia and New Zealand stock exchanges list long-dated equity participations where the investor takes the full dividend risk.

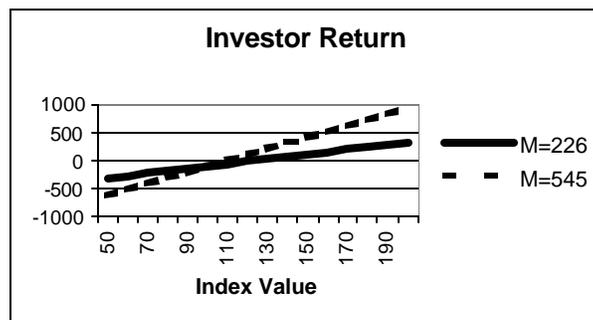


Figure 3: Investor's return as function index value at maturity.

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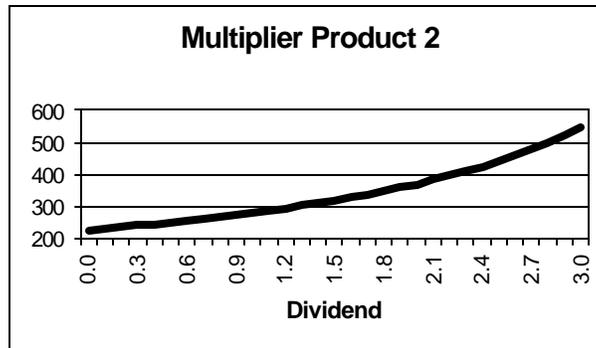


Figure 4: Multiplier as function annual dividend

These are known as ‘endowment warrants’.<sup>3</sup> At initiation, the investor pays an upfront amount which depends on interest rates and the relevant stock’s expected dividend yield. The difference between this amount and the actual stock price is referred to as the initial ‘outstanding amount’. During the life of the participation the outstanding amount grows at the interest rate and shrinks with the dividends paid by the stock. At maturity there are two possible outcomes. Either the dividends received have been sufficient to reduce the outstanding amount to zero or not. In the first case the investor receives the value of the stock. In the first case the investor receives the value of the stock. In the second case the investor receives an amount equal to the difference between the value of the stock and what is left of the outstanding amount. In other words, the investor pays what is left of the outstanding amount and the derivatives firm pays the value of the stock.

Instead of leaving the dividends with the derivatives firm the derivatives firm and the product provider can of course also pass them on to the investor as they come in. In order to obtain a favourable tax treatment of the product as a whole at the investor's end it may well be necessary to pass them on. We will discuss this in more detail later

#### **IV Protected Equity-Linked Saving**

In the previous sections we saw that an appealing level of equity exposure can only be obtained at the cost of higher downside risk for the investor as well as for the product provider. It is therefore worthwhile to look for ways to limit the downside risk. One way to do so is to give the investor the right to cancel the contract at maturity. Since he will do so only if he has to pay, this means offering the investor a payoff at maturity equal to

$$X_5 = M \times \text{Max} [0, I_5 - I_0] \quad (4)$$

With a payoff like this the investor receives an amount equal to a multiple of the change in the index value over the product's life but only if the latter is positive. In return the investor makes the usual 60 monthly payments of 100 to the product provider. The latter takes 5.62 out of every 100 for himself and pass the remainder on to the derivatives firm that provides the required payoff. Since  $\text{Max} [0, I_5 - I_0]$  is nothing more than the payoff of an ordinary at-the-money call option, the budget equation is given by

$$5,000 = M \times C_0[I_0, 5] \quad (5)$$

where  $C_0[K,T]$  is the derivatives firm's offer for an ordinary call with strike price  $K$  and time to maturity  $T$ . The derivatives firm prices the option at 29.14. The multiplier the product provider can offer the investor is therefore equal to 172. Ordinary call options become cheaper if the reference index pays dividends. This means that the participation rate of product 3 can again be raised by picking a reference index with a high dividend yield. With an annual dividend yield of 3% the option price comes down from 29.14 to 18.96. As a result, the multiplier goes up from 172 to 264.

## **V The Structurer's Perspective**

So far we have assumed the product provider hedges himself by entering into a swap with a derivatives firm. This is not the only possibility though. Since hedging an index participation boils down to nothing more than buying the index, the product provider could easily cut the derivatives firm out and hedge product 1 himself. He could sell the stream of 60 monthly payments he receives from the investor in the market and use the proceeds to buy the index. At maturity he would then liquidate the position and pay the investor. Although we speak of 'selling the 60 monthly payments from the investor in the market' this is of course nothing more than borrowing. The amount the product provider borrows is such that the 60 monthly payments by the investor are exactly enough to pay the interest as well as redeem the loan at maturity.

Product 2 can be hedged in the same way except that the size of the loan increases. In this case the monthly payments made by the investor are no longer enough to cover

both interest and redemption which is why the investor has to pay an additional lump sum at maturity. We refer to this form of leverage as **aggressive leverage** as opposed to **conservative leverage** where the investor's monthly payments equal the combined payment of interest and redemption. Aggressive leverage can range from giving the investor a little more exposure than with conservative leverage to giving him a lot more. Although the distinction between interest and redemption is artificial, we can think about it as follows. Starting with a conservatively leveraged position, if the size of the loan increases the redemption component in the investor's monthly payments shrinks and starts building up in the terminal lump sum. If the size of the loan continues to increase we reach a point where the investor's monthly payments become pure interest. At this point the terminal lump sum equals the full size of the loan, i.e.  $N_5 = M \times I_0$ . Increasing the loan beyond this point means that apart from redemption, the terminal lump sum also contains an interest component.

If the multiplier in product 2 is chosen such that the  $N_5 = M \times I_0$  investor receives a payoff equal to  $M \times (I_5 - I_0)$ . Looking at this payoff more closely, we see that it is nothing more than  $M$  times the amount the investor would have received if he had entered into a forward contract where he paid the starting value of the index and received the index value at maturity. We can therefore interpret product 2 not only as the aggressively leveraged purchase of the index but also as the conservatively leveraged purchase of  $M$  off-market forwards. Product 3 can be interpreted in a similar way. From expression (5) it is clear that the payoff of product 3 can be obtained by the conservatively leveraged purchase of ordinary calls. The same payoff, however, can also be obtained by the aggressively leveraged purchase of a principal note with a 0% guaranteed minimum return. This shows that conservative

leverage is not necessarily less risky than aggressive leverage. It all depends on how much implicit leverage there is in the assets that are purchased with the proceeds of the loan. Table 1 summarizes the payoffs of the three equity linked savings products which we discussed using the concept of leveraged buying.

## VI The Marketer's Perspective

Knowing how to hedge and price equity-linked savings products, the next question is how to present them to the public. Let's look at product 1 and 2 first. Whether he

<b>Asset Bought</b>	<b>Conservative Leverage</b>	<b>Aggressive Leverage</b>
Stocks	Product 1	Product 2
Forwards	Product 2	
Protected notes		Product 3
Call options	Product 3	

Table 1: Overview equity linked savings products

Enters into a back-to-back swap with a derivatives firm or not, the product provider acts as a swap provider to the investor. The investor periodically pays a fixed amount and the product provider in return provides a payoff equal to the value of a certain number of stocks (minus a fixed amount). Building on what we discussed in the previous section, however, the product provider can also present himself as a financial intermediary that offers investors loans and at the same time arranges for the purchase

of equity with the proceeds of those loans. He can for example explain product 1 by saying that at  $t = 0$  he lends the investor 5,000 and invests the proceeds in equity for him. The loan is paid for and redeemed by 60 monthly payments of 100. At maturity the investor is debt-free and the stocks are his. The implied interest rate on the loan is 7.4%. A similar story can be told for product 2. At  $t = 0$  the investor borrows 22,600 from the product provider and the proceeds are invested in equity. The interest on the loan is paid in the form of 60 monthly payments of 100. At maturity the stocks are sold and the investor redeems the loan. If the stocks are worth more than 22,600 he makes a profit and otherwise he shows a loss. The implied interest rate is 5.3%. This is lower than for product 1 because the amount borrowed is higher while the product provider's profit is unchanged.

Depending on the fiscal environment, presenting product 1 and 2 as leveraged equity purchases may yield significant tax advantages. In the Netherlands retail investors are allowed to deduct the interest paid on loans used to buy equity with from their taxable income. With a 60% marginal income tax rate this makes leveraged products where the interest paid is readily observable very attractive. With product 2 the monthly payment would be fully tax deductible, meaning that after tax the investor only pays 40 per month instead of 100. The After-tax implies interest rate is therefore only 2.1%. If the index rises by 60% the investor makes an after-tax return of 540%. If it rises by 100%, he makes 967%.

To obtain favourable tax treatment it may be necessary to physically deliver the stocks to the investor at maturity. This serves to emphasize to the authorities that the investor is doing nothing more than buying stocks with borrowed money. It may be

necessary to pass on the dividends. This need not be restrictive, however, as the product provider can get them back by requiring the investor to pay an identical amount of additional 'premium' for some reason. We will encounter an example of this in the next section. Thanks to the favourable tax treatment and the continuing bull market, equity-linked savings products have become very popular in The Netherlands, with the notional outstanding now in the billions of dollars. There is a downside to this popularity, however. The Dutch IRS will change the taxation of leveraged equity as of January 1<sup>st</sup>, 2001.

Product 3 can be presented in a similar way. The investor borrows 17,000 from the product provider and the proceeds are invested in equity. This is identical to product 2. With product 3 however, the product provider will make up for the loss in case the stocks end up worth less than 17,200. The implied interest rate is 7.0%. This is higher than with product 2 not only because the amount borrowed is lower but also because the investor now has to pay for the guarantee. Apart from the possible tax benefits, presenting product 3 in this way offers significant marketing opportunities. Who would not want to invest with a friendly financial intermediary that not only provides cheap loans and arranges for the purchase of equity at zero cost, but which also guarantees the investor will never lose?

From a marketing perspective it is interesting to experiment a little more with different ways of presentation. Let's look at product 3 for example. Instead of saying that the investor will be paid a multiple times the positive change in the index, the product provider can express this payoff in terms of a notional amount  $N$ , a participation rate  $\alpha$  and the index return by rewriting (4) as

$$X_5 = N \times \left( \text{Max} \left[ 0, \alpha \left( \frac{I_5 - I_0}{I_0} \right) \right] \right) \quad (6)$$

which of course is the same as

$$X_5 = \frac{\alpha N}{I_0} \times \text{Max} [0, I_5 - I_0 ] \quad (7)$$

We now have two parameters to choose:  $\alpha$  and  $N$ . Since nothing has changed to the payoff, however, both parameters have to be set such that  $\alpha N / I_0 = 172$ . Making a choice, one has to keep in mind how the investor will react. If the product provider opts for a high notional amount and a low participation rate the investor may think he is getting a bad deal. From a marketing perspective it is therefore better to go for a low notional amount and a high participation rate. One alternative is to make the notional amount equal to the total amount paid by the investor of the product's life, i.e. 6,000. This yields a participation rate of little over 2.87. In other words, the product provider would pay the investor 2.87 times the positive index return over a notional amount of 6,000. Compared to the much lower levels of participation offered on principal protected notes, which hover between 0.6 and 0.8, this looks like a little miracle. Unfortunately, product 3 is not a principal protected note with a 0% guaranteed minimum return will pay off at least 6,000 at maturity. With product 3, however, the investor is only protected against losing more than the amount that was put in.

Another interesting way to present the above products is to split the payoff into two parts: (1) a regular payoff and (2) a money-back payment. Although there is absolutely no difference in the way these two parts are calculated, the product provider present them in a completely different way. With product 3 the investor gets paid 172 times the positive change in the index. The product provider need not present the product in that way though. He could say that the actual product only paid off 112 times the change in the index and that the remaining 60 times the index resulted from a special money-back feature that paid back part or eve all of the money paid by the investor if the index went up. With the investor paying 6,000 in total and the index initially at 100, he could say that the investor gets back 1% of what he pays for every 1% that the index goes up. If over the product's life the index went up from 100 to 140 the product would pay off  $72 \times 40 = 6,880$  of which  $112 \times 40 = 4,480$  would be considered regular payoff and  $60 \times 40 = 2,400$  would be considered money paid back. If the index went up to 200 the money back part would be 6,000. If it went above, the money-back payoff would be even higher. Since this is clearly undesirable, the money-back part will therefore have to be capped at 6,000. This can easily be done by writing 60 ordinary calls with strike 200. The premium received from doing so can be used to increase the multiplier on the regular payoff.

## **VII The Investor's Perspective**

So far we have first looked at what the investor pays in, taken out some profit for the product provider can offer given the desired profit margin and the derivative firm's pricing. Looking back at product 1, 2 and 3 we could, however, ask ourselves whether at these prices we are not offering more than the investor expect. If this is the case the

product provider can take out more without losing volume. And even if he would lose some volume, since he makes more he can afford to do a little less.

What would the investor consider a fair deal? There are several ways to answer this question. One way is to look at the **implied interest rate** on a product and compare that with the interest rate the investor could borrow at himself. The implied interest rates for product 2 and 3 are shown in figure 5 and are very interesting. With the product provider taking out 5.62 of every 100 the implied interest rate is only 5.3% for product 2 and 6.9% for product 3. From the graph we see that even if the product provider took 30 out of every 100 the implied interest rate would not rise higher than 7.1% for product 2 and 4.9% for product 3. With wholesale rates at 5% investors will not be able to borrow at these rates themselves, especially since they use the proceeds to buy equity with. This strongly suggests that the product provider could take out a lot more than 5.62.

We can also look at the product payoffs in **probabilistic terms**. Typically people think a gamble is fair if there is a 50-50 chance of winning and losing. So let's see what product 2 and 3 have to offer in these terms. Figure 6 shows the probability of a payoff lower than the present value of the payments made by the investor, assuming an unexpected annual index return of 12% and an annual index volatility of 20% (a conservative estimate of today's retail investor's expectations). Taking out 5.62 per month these probabilities would only drop to 74% and 66% respectively. In other words, taking out 30 instead of 5.62 still allows for a very impressive upside, again strongly suggesting that the product provider could take out more than 5.62. Let's look at product 3 in more detail. Taking out 30 per month the multiplier will drop

from 172 to 127. If the index return is negative the investor loses his 60 monthly payments of 100. However, the probability of this happening is only 9%. As mentioned, the probability of getting back less than he put in is only 34%. The probability of at least doubling his money would be 31% and the probability of at least tripling 7.33%. If the product provider was able to sell the product at these conditions it would make an upfront profit of 1,589. Definitely not a bad result for selling 127 ordinary call options back-to-back.

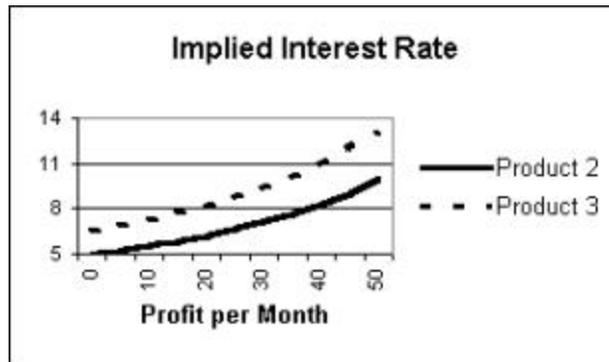


Figure 5: Implied interest rate as function monthly profit margin.

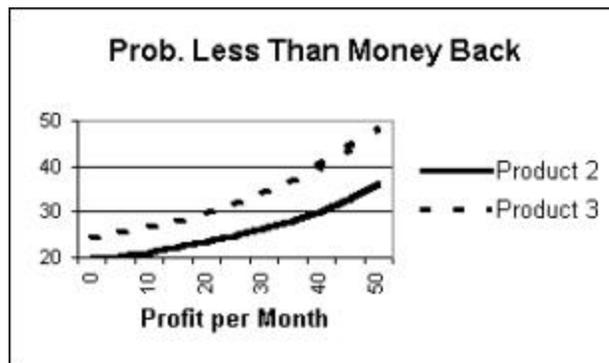


Figure 6: Probability of payoff less than 5,298 as function monthly profit margin.

## **VIII A Real-Life Example**

One product recently offered in The Netherlands required investor to pay 250 per month for three years. In return, the product provider lent the investor 42,893 which was then invested in a basket of three different high yielding Dutch stocks. Dividends are passed on to the investor at first but then paid back to the product provider in the form of an extra 'premium'. The way this premium is justified is quite interesting. The product provider presents the purchase of the above stocks as a three-step procedure. At  $t = 0$  14,298 worth of stocks is bought. At  $t = 1$  another 14,298 worth of stocks are bought and the same happens at  $t = 2$ . According to the product provider the extra premium serves to ensure the investor that the stocks bought at  $t = 1$  and  $t = 2$  are, instead of at the prevailing prices, bought at  $t = 0$  prices. In reality, all stocks are of course bought at  $t = 0$ . The above story simply serves as an excuse to get the dividends back from the investor after first passing them on to emphasize to the authorities that the investor is really buying equity. At maturity the investor receives the value of his stocks minus the amount initially borrowed.

Now let's see what the product provider is taking out of the deal. Identifying the basket as the relevant index, the payoff of the above product can be expressed as

$$X_3 = M \times I_3 - 42,893, \quad (8)$$

where  $M = 42,89 / b$ . With the product provider being part of a large AA rated financial institution it is able to fund itself at 4.0% (at the time of issuance). The present value of the 36 coupons it receives is therefore 8,467. After taking out a profit this has to pay for the payoff given by expression (9). The first part of the latter payoff can be hedged by buying  $M$  shares in the index and a price  $I_0$ . Since the investor has

to pay back the gross dividends which he receives, the product provider effectively keeps the dividends. With a relatively high dividend yield on the basket of 2.8% this brings in at least 100 per month, which has a present value of 3,387. To buy  $M$  shares in the index the product provider therefore needs to invest only 39,506. The second part of the payoff can be hedged by borrowing the present value of 42,893, which is 38,043. Together this means that the product provider will be able to generate the required payoff at a cost of only 1,463. However, in total he receives an amount of 8,467, implying a stunning profit of 7,004. Making a profit of 7,004 on a loan of 42,893 may seem outrageous but it should be emphasized that the marketing and operational costs of retail products can be very high. Part of the above margin should therefore be seen as compensation for these costs and not as outright profit.

The next question is why Dutch retail investors buy into equity-linked savings products at these very high prices? Initially it was thought that the only reason was the tax advantage. When the authorities announced their plans to abolish the tax deductibility of the interest paid many feared the product would lose its appeal. Demand for the equity-linked savings products remained strong, however. There are a number of reasons for this.

- After missing out on a couple of years of exceptionally strong equity market performance, investors simply want the exposure. Not only because they think markets will rise further, but also because they need to keep up with their neighbours. What is worse in today's society than to see your neighbour make money by doing something you could easily have done yourself as well? This

also explains the popularity of product 2, which offers higher exposure than product 3.

- Even with the product provider taking out a lot the expected payoff is still quite fascinating, especially at the highly optimistic scenarios typically used to market these products.
- The product is acquired by paying relatively small amounts over a long period of time. In many investors' minds it is therefore probably not more than an interesting gamble at the side of a serious investment.
- It has been argued that for retail investor equity-linked savings products are the only way to acquire equity exposure. Although this is true for the ones that are not able to pay upfront, marketing research suggests that many buyers do have sufficient cash available.
- Not many retail investors know enough about derivatives to figure out that they could replicate the payoff of product 2 by buying calls and selling puts or replicate product 3 by simply buying call options in the listed market.

In sum, for product providers issuing equity-linked savings products is attractive because retail investors are generally short on alternatives as well as sophistication and sometimes tend to combine that with less rational behaviour. Thanks to the bull market, it has worked out very well for all parties involved so far.

## IX Variations in Protected Equity-Linked Saving

We can take equity-linked savings a lot further. Concentrating on product 3, we could allow for a guaranteed minimum return other than zero, we could introduce a cap as well as more exotic bells and whistles. After all, the protected equity-linked savings product is not different from conservatively leveraged call option or an aggressively leveraged principal protected note. In addition, we need not settle for one type of option. We can divide the money available over two or even more different options.

### A A Disappointment Bonus

One interesting possibility is to create a product that offers investors a payoff equal to

$$X_5 = M \times \left( \text{Max}[0, I_5 - I_0] + (I_0 - M_{0,1}^-) \right) \quad (9)$$

where  $M_{0,1}^-$  denoted the lowest value of the reference index over the first year of the product's life. This product pays the investor a multiple of the positive change in the index plus a 'bonus' equal to the same multiple times the difference between the initial index value and the lowest index value over the first year of the product's life. This bonus is obtained even if the reference index at maturity is below its starting value. Looking more closely at the above payoff we see that the product provider is selling the investor a combination of an ordinary call and a fixed-strike lookback put. The budget equation for the swap which would support a product like this is therefore given by

$$5,000 = M \times (C_0[I_0, 5] + LBP_0[I_0, 0, 1, 5]) \quad (10)$$

where  $LBP_0[K, t_1, t_2, T]$  is the derivatives firm's offer for a fixed-strike lookback put with strike  $K$ , monitoring from  $t = t_1$  until  $t = t_2$  and time to maturity  $T$ . Using the results on partial lookback options in Heynen and Kat (1994), the derivatives firm prices the options package at 57.15, implying that the product provider can offer the investor a multiplier of 106.

If we wanted to disguise the true mechanics of the structure a little better we could rewrite the above payoff as

$$X_5 = N \times \left( \text{Max} \left[ 0, \mathbf{a} \left( \frac{I_5 - I_0}{I_0} \right) \right] + \mathbf{a} \left( \frac{I_0 - M_{0,1}^-}{I_0} \right) \right) \quad (11)$$

As discussed before, the product provider is free to choose the notional amount  $N$  and the participation rate  $\mathbf{a}$  as he sees fit under the restriction that  $\mathbf{a} N / I_0 = 106$ . If he would choose a notional amount of 6,000, he would get a participation rate of 1.77. If the reference index went up the product provider would therefore pay the investor.

$$X_5 = 6,000 \times 1.77 \times \left( \left( \frac{I_5 - I_0}{I_0} \right) + \left( \frac{I_0 - M_{0,1}^-}{I_0} \right) \right) \quad (12)$$

Over a notional amount of 6,000, the product would pay the investor 1.77 times the index return plus on top of that a bonus in case the index came below its starting value during the first year. This sounds too good to be true but once again one has to realize that this is not a principal protected note. The only protection offered is that the investor will not lose more than he put in.

With the above product the bonus has the same multiplier (or participation rate) as the part of the payoff related to the index return. This is not necessarily though. The product provider could structure a product that only paid the investor half the above disappointment bonus. This would yield a payoff equal to

$$X_5 = M \times \left( \text{Max}[0, I_5 - I_0] + 0.5 \times [I_0 - M_{0,1}^-] \right) \quad (13)$$

The budget equation for this payoff is given by

$$5,000 = M \times (C_0[I_0, 5] + 0.5 \times LBP_0[I_0, 0, 1, 5]) \quad (14)$$

The derivatives firm prices the above options package at 38.14. This means that in this case the product provider can offer the investor a multiplier of 131.

In our more general framework of notionals and participation rates the payoff at maturity would be

$$X_5 = N \times \left( \text{Max} \left[ 0, a \left( \frac{I_5 - I_0}{I_0} \right) \right] + 0.5 \times a \left( \frac{I_0 - M_{0,1}^-}{I_0} \right) \right) \quad (15)$$

where the notional and the participation rate have to be set such that  $= 131$ . If the participation rate was set equal to 2 the notional amount would be equal to 6,550. In case the index went up this would leave the investor with a payoff equal to

$$X_5 = 6,500 \times \left( 2 \times \left( \frac{I_5 - I_0}{I_0} \right) + \frac{I_0 - M_{0,1}^-}{I_0} \right) \quad (16)$$

Over a notional amount of 6,550, this product pays the investor 2 times the positive index return plus a bonus in case the index goes down during the first year. The bonus, however, has an effective participation rate equal to 1 instead of 2. Although this may seem a rather strange payoff to offer retail investors, it allows for an appealing story. The product provider can explain this payoff to investor by saying that by paying 100 every month for five years they will get protected exposure to 6,550 worth of equity and that if the index indeed goes up the product provider will double their profits. In addition, if the index goes down in the first year, the product provider will make up for their loss. Alternatively, we could of course say that investors will get protected exposure to 13,100 worth of equity and that if the index goes down in the first year the product provider will make up half their loss.

## **B A Partial Cap**

Another interesting product arises if the money available is divided over an ordinary call and a call spread. In that case the payoff to the investor a payoff would be equal to

$$X_5 = N \times \left( \text{Max} \left[ 0, \mathbf{a} \left( \frac{I_5 - I_0}{I_0} \right) \right] + \text{Min} \left[ H, \text{Max} \left[ 0, \mathbf{a} \left( \frac{I_5 - I_0}{I_0} \right) \right] \right] \right) \quad (17)$$

The budget equation for a product with the above payoff is given by

$$5,000 = \frac{\mathbf{a}N}{I_0} \times \left( C_0[I_0, 5] + CS_0 \left[ I_0, I_0 \left( 1 + \frac{H}{\mathbf{a}} \right) 5 \right] \right) \quad (18)$$

where  $CS_0[K^1, K^2, T]$  is the derivatives firm's offer for an ordinary call spread with lower strike  $K^1$ , upper strike  $K^2$  and time to maturity  $T$ . Expression (18) shows clearly

that by selling the investor this product the product provider is doing nothing more than selling him ordinary call options with strike  $I_0$  as well as ordinary call spread with lower strike  $I_0$  and upper strike  $I_0(1 + H/a)$ .

Before we can solve the budget equation we need to fix the cap rate  $H$  and either  $N$  or  $a$ . Suppose we fixed the cap rate at 60% and the participation rate at 1. For the short call in the call spread this implies a strike of  $1.6I_0$ . Knowing the strike, the derivatives firm prices the options package at 48.41, meaning that the notional amount is going to be 10,328. The result is again quite interesting. By paying 60 every month during five years the investor obtains protected equity exposure in the amount of 10,328 and if the index rises the product provider doubles the investor's profit up to a maximum of 6,197 ( $= 0.6 \times 10,328$ ).

## **C Money-Back Structures**

The money-back structure discussed earlier was created by simply splitting up the original product payoff in a standard part and a money-back part. The product provider can, however, also include other types of money-back features. Suppose he equipped product 3 with a single barrier money-back feature, on top of the regular payoff, the investor receives a prefixed amount  $B$  if during the life of the product the index goes up by more than  $X\%$  from its value at initiation. This yields a payoff function equal to

$$X_5 = M \times \text{Max}[0, I_5 - I_0] + D \times B, \quad (19)$$

where

$$\begin{aligned} D &= I, & \text{if } \exists_j I_j \geq I_0(1+X), \\ &= 0, & \text{if } \forall_j I_j > I_0(1+X). \end{aligned}$$

The subscript  $j$  counts the monitoring points, i.e. the points in time where we check for a barrier hit. With the investor paying 6,000 in total, the product provider could set  $B = 3,000$ , meaning the investor would get half of his money back if at any of the monitoring points the index was higher than  $I_0(1+X)$ . If he set  $X = 0.6$  the money-back feature would be worth 946 which in turn would yield a multiplier of 139.

## D Segmented Payoff Functions

In none of the payoffs discussed so far do we find any reference to the way the investor pays for them. Instead of paying 60 monthly instalments of 100, the investor might as well the product provider the present value of that upfront. If the product provider wanted a product where the payoff relates directly to the 60 monthly payments which the investor makes he could offer investors a payoff of

$$\begin{aligned} X_5 = 100 \times & \left( \text{Max} \left[ 0, \mathbf{a} \left( \frac{I_5 - I_0}{I_0} \right) \right] + \text{Max} \left[ 0, \mathbf{a} \left( \frac{I_5 - I_{1/12}}{I_{1/12}} \right) \right] \right. \\ & \left. + \dots + \text{Max} \left[ 0, \mathbf{a} \left( \frac{I_5 - I_{59/12}}{I_{59/12}} \right) \right] \right) = 100 \mathbf{a} \sum_{i=0}^{59} \text{Max} \left[ 0, \left( \frac{I_5 - I_{i/12}}{I_{i/12}} \right) \right]. \end{aligned} \quad (20)$$

This payoff is made up of 60 parts, each of which can be thought of as  $t = 5$  payoff bought with 100 at the end of the month  $i$ . Every one of these payoffs is equal to

$$X_5^i = 100\mathbf{a} \times \left( \frac{1}{I_{i/12}} \times \text{Max} \left[ 0, I_5 - I_{i/12} \right] \right), \quad i = 0, 1, \dots, 59, \quad (21)$$

which is the same as the payoff of  $100\alpha$  5-year quantity-adjusting forward-starting call options with multiplier  $1/ I^{***}$  and the strike price  $^{****}$ . We can therefore find the participation rate that the product provider can offer investors from the following equation

$$5,000 = 100\mathbf{a} \times \sum_{i=0}^{59} QFC_0^i \left[ \frac{1}{I_{i/12}}, I_{i/12}, 5 \right] \quad (22)$$

where  $QFS_0[M,K,T]$  denotes the derivatives firm's offer price for a quantity-adjusting forward-starting call with multiplier  $M$ , strike  $K$  and settlement time at  $T$ . Of course, if  $i = 0$  we simply have an ordinary call option. Using the results of Heynen and Kat (1995) the derivatives firm quotes a price of 9.68 for the package. This yields a participation rate of 5.17. Compared to the participation rate of 2.87 that we found earlier this may look high, but we should keep in mind that in this case the investor only gets paid the change in the index measured from the moment he makes his payments and not from  $t = 0$ .

The above product is different from a buy-as-you-go scheme, which buys additional call options when the investor makes his next payment. With the latter strategy we do not know in advance how many options we will be able to buy since we do not know the future option prices. With the above product, however, we are able to fix everything from the outset because we do not buy the options over time. We buy them all on the starting date using conservative leverage.

## **X Variations in Investor Payment**

Until now we have assumed the investor pays 100 every month for the next five years. This is the most obvious and straightforward choice. One can, however, arrange for payment in various other ways as well. The amounts paid can vary deterministically, for example by instead of 100 only taking 25 during the summer holiday season and in December, or they can be linked with a reference table. One interesting thought is to attach knock-out conditions to the payments to be made by the investor. The product provider could say that after three years the reference index was up by more than a certain percentage the investor would not have to make any more payments. Clearly this means adding a European knockout barrier to the payments to be made in year 4 and 5: if at  $t = 3$  the index is higher than the barrier, all year 4 and 5 payments knock out. If this was thought too rigorous one might consider tranching the payments made by the investor and giving each tranche its own knockout barrier. Suppose the 100 to be paid was split in five tranches of 20 each. If we then equipped all five tranches with a knockout barrier at levels  $H^1$  up to  $H^5$  respectively we would have created a payment that knocked out stepwise. If no barrier was hit, the payments for year 4 and 5 would stay at 100. If only  $H^1$  was hit, the monthly payment would drop to 80. If only  $H^1$  and  $H^2$  were hit, it would drop to 60 etc.

With a knockout condition the payments to be made to the investor change from fixed cash flows to digital cash flows. The latter will be worth less than the fixed payments the investor used to make and the multiplier will therefore have to come down. Let's look at product 3 again. With product 3 the investor pays 100 for 60 months of which

the product provider takes off 5.62 every month. Under the assumptions made, the present value of the investor's payments to the derivative's firm is 5,000. With the required call option costing 29.14 the multiplier is 172. Now suppose we added a European knockout barrier to the payments to be made by the investor in year 4 and 5. If at  $t = 3$  the index was higher than 140, all year 4 and 5 payments would knockout. Assuming the product provider wanted to make the profit the same as before, in this case it will be easier for the product provider to simply pass all the investor's payments through to the derivative's firm and ask the latter to make an upfront payment of 298 or provide a stream of 60 monthly payments of 5.62. The derivatives firm values the first three years of payments at 3,336 and, using the results of Rubenstein and Reiner (1991), the second two years of digitals at 1,495. Paying the product provider 298 upfront this leaves 4,533 for the call options. With an option worth 29.14, this means we can now offer the investor a multiplier of 155 instead of 172.

In comparison with the standard product, many investors will consider this a good deal. What happens here is that the product provider exploits the difference between the investor's return expectation, which is largely based on recent market performance, and the distribution used by the derivatives firm to price the digitals involved. Assuming an expected return of 12% and 20% volatility, the investor attributes a 45% probability to the index being higher than 140 in three years time. For pricing purposes, however, the derivatives firm will assume this probability to be only half that. In other words, in the eyes of the investor the derivatives firm pays a very high price for something which he thinks is worth 30% less.

## **XI Conclusion**

In this paper we showed how equity-linked savings products are structured, priced and marketed. A typical equity-linked savings product is made up of two parts. The first part is a stream of periodic payments made by the investor. This is the savings part. The second part consists of a single payment at maturity which is linked to the behaviour of the reference index or indices in question. This is the equity-linked part.

Technically, equity-linked savings products are swaps. From a hedging perspective, however, we can think of these products as leveraged purchases of forwards and/or options. Another angle appears when marketing equity-linked savings products. Instead of presenting them as swaps or as leveraged purchases of forwards and/or options, they may also be presented as the leveraged purchase of stocks (with additional bells and whistles). Apart from making these products easier to explain to retail investors, depending on the fiscal environment, this may bring significant tax advantages as demonstrated by the case of the Netherlands.

Our analysis shows that, contrary to popular belief, it is very well possible to sell highly exotic derivatives to retail investors. In fact, the majority of exotic equity structures ends up in the hands of retail instead of institutional investors, either in the form of equity-linked notes or equity-linked savings products. In other words, in equity derivatives the most sophisticated products tend to be sold to the least sophisticated clients.

When it comes to pricing equity-linked savings products we can take the desired margin as given and derive the conditions we can offer, or we can first decide on the conditions to offer and derive the maximum margin from there. Following the latter approach, it appears that equity-linked savings products can offer product providers extremely high margins. This confirms what retail bankers, (life)-insurers and many others discovered many years ago: it is hard to be too greedy in the retail business.

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